OPERATING INSTRUCTIONS

for

TYPE 821-A TWIN-T IMPEDANCE-MEASURING CIRCUIT

Form 566-D

GENERAL RADIO COMPANY

CAMBRIDGE 39 MASSACHUSETTS

NEW YORK CHICAGO LOS ANGELES

U. S. A.

TABLE OF CONTENTS

OPERATING INSTRUCTIONS
FOR
TYPE 821-A TWIN-T IMPEDANCE-MEASURING CIRCUIT

1.0 DESCRIPTION

1.1 General Description

The Twin-T Impedance-Measuring Circuit is a null instrument for use in measuring impedance at frequencies from 460 kc to 30 Mc. Measurements can be made at frequencies slightly below and above this nominal frequency range.

It is used basically with a parallel-substitution method for measuring unknown impedances in terms of their parallel admittance components, namely susceptance, B, and conductance, G.[1] The susceptance is obtained from capacitance increments, read from a dial directly calibrated in capacitance (in µµf), by means of the relation:

$$B = \omega \Delta C \qquad (1)$$

The conductance is obtained from a dial directly calibrated in conductance (in µmho). Conversion from the parallel admittance components, G and B, to series impedance components, R and X, can be made, if desired, through the relations:

$$R = \frac{G}{G^2 + B^2} \qquad (2)$$

$$X = \frac{-B}{G^2 + B^2} \qquad (3)$$

1.2 Circuit and Balance Conditions

The circuit used consists of two T networks connected so that they furnish parallel transmission paths, a-b-c and a-d-c, from a high-frequency generator to a null detector as shown in Figure 1.

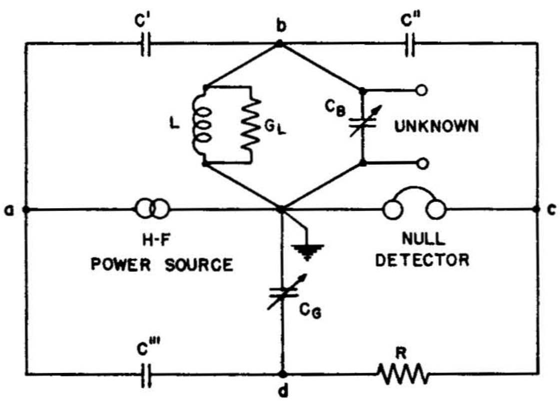

FIGURE 1. Basic diagram of Twin-T

Zero energy transfer from the generator to the detector occurs when the transfer impedances[2] of the two T networks are made equal and opposite, and a null balance is obtained. The circuit conditions for which this occurs are expressed by

$$G_L - R\omega^2 C'C'' \left(1 + \frac{C_G}{C'''}\right) = 0 \qquad (4)$$

$$C_B + C'C''\left(\frac{1}{C'} + \frac{1}{C''} + \frac{1}{C'''}\right) - \frac{1}{\omega^2 L} = 0 \qquad (5)$$

1.3 Method of Measurement

In measuring an unknown admittance, $Y_X = G_X + jB_X$, the circuit is initially balanced to a null. The unknown admittance is then connected to the UNKNOWN terminals and the circuit rebalanced by adjusting the conductance condensers, C_G, and the susceptance condenser, C_B. From the initial capacitance values, C_{G_1} and C_{B_1}, and the final capacitance values, C_{G_2} and C_{B_2}, the unknown admittance components are found as follows:

$$G_X = \frac{R\omega^2 C'C''}{C'''}\left(C_{G_2} - C_{G_1}\right) \qquad (4a)$$

$$B_X = \omega\left(C_{B_1} - C_{B_2}\right) \qquad (5a)$$

These relations show that each of the components is proportional to a capacitance increment. Since the unknown conductance component is always positive, the capacitance of the conductance condenser, C_G, must always be increased when an unknown impedance is connected to the UNKNOWN terminals and a single scale can be provided reading incremental conductance from 0 µmho, at the minimum capacitance value, to a value determined by the capacitance range. The susceptance component, on the other hand, may be either positive or negative. For direct readings, therefore, two scales would be necessary, one

1. The convention Y = G + jB is used throughout so that capacitive susceptance is considered positive, inductive negative.

2. Defined as the ratio of the input voltage to the output current when the output terminals are short-circuited.

reading from 0 µmho at maximum capacitance and the other from 0 µmho at minimum capacitance. To avoid confusion from the use of two scales, reading in opposite directions, a single scale of absolute susceptance can be substituted, the unknown susceptance being found from the difference between the two absolute susceptances at balance, as indicated in equation (5a).[3]

1.4 Conductance Range

For a given conductance value the capacitance increment, $C_{G_2} - C_{G_1}$, varies inversely as the square of the frequency, as shown in equation (4a). The conductance dial, therefore, reads directly only at a single frequency and, at all other frequencies, the reading must be multiplied by the square of the ratio of the operating frequency to the frequency at which the dial was calibrated.

In the Twin-T Impedance-Measuring Circuit the use of a single conductance scale is not feasible because of the very large change in conductance range that results from the wide frequency band covered. To prevent this excessive variation of conductance range the multiplying factor $\frac{R\omega^2 C'C''}{C'''}$ in equation (4a) is changed by adjustment of the condensers C' and C". By switching these condensers, for instance, a single scale could be made direct-reading at several different frequencies and the variation in conductance range at frequencies between these could be made reasonably small. Some increase in range, as the frequency increases, seems desirable, however, from a consideration of the frequency characteristics of common types of circuit elements.[4] The Twin-T has, therefore, been provided with a four-position switch that establishes linearly increasing conductance ranges at successively higher frequencies, as follows:

Nominal Switch-Position Frequency	Conductance Range
1 Mc	0 to 100 µmho
3 Mc	0 to 300 µmho
10 Mc	0 to 1000 µmho
30 Mc	0 to 3000 µmho

To accommodate these on the dial, two scales are provided, one reading from 0 to 100 µmho and one from 0 to 300 µmho. The first scale is read directly at 1 Mc, the second at 3 Mc. The first is again used, with a multiplying factor of 10, at 10 Mc and the second, with a multiplying factor of 10, at 30 Mc. At other frequencies the dial reading corresponding to a given nominal switch-position frequency must be multiplied by the square of the ratio of the operating frequency to the nominal switch-position frequency.

1.5 Susceptance Range

For a given susceptance value, the capacitance increment, $C_{B_1} - C_{B_2}$, varies inversely as the frequency, as shown in equation (5a). A susceptance dial, therefore, would read directly only at a single frequency. Since, in many cases, the effective parallel capacitance is as convenient a quantity to measure as the susceptance, and since capacitance does not vary with frequency, the Twin-T has been provided with a dial calibrated in capacitance rather than susceptance. For the reasons outlined in paragraph 1.3 it reads directly the absolute capacitance, rather than incremental capacitance. It has a range from 100 µµf to 1100 µµf and can therefore be used directly to measure effective parallel capacitances from -1000 µµf to +1000 µµf. At the nominal switch-position frequencies, this range of effective parallel capacitance corresponds to the following susceptance ranges:

Nominal Switch-Position Frequency	Susceptance Range
1 Mc	-6,280 to +6,280 µmho
3 Mc	-18,840 to +18,840 µmho
10 Mc	-62,800 to +62,800 µmho
30 Mc[5]	-188,400 to +188,400 µmho

1.6 Auxiliary Controls

Equation (4) shows that the setting of the conductance condenser, C_G, for the initial conductance balance is determined by the effective conductance, G_L, of the tuning coil, L. Since this conductance

3. For the reason outlined in paragraph 1.5 the dial used with the Twin-T is calibrated in absolute capacitance, rather than susceptance.

4. For instance, the conductances of coils that are tuned with the same variable condenser over different wave bands and that have similar values of Q will increase with frequency, as will those of condensers and dielectric samples having reasonably constant power factors.

5. Because of errors caused by residual parameters, discussed in Section 3.2, the full range of the condenser cannot be used above 20 Mc. At 30 Mc the actual usable capacitance increment is about 300 µµf and the corresponding susceptance range from -56,500 to +56,500 µmho.

does not, in general, vary as the square of the frequency[6] the initial setting of the conductance condenser will change with the frequency. In order to avoid this variation and to take full advantage of the calibrated conductance scale, an auxiliary condenser is connected in parallel with the conductance condenser. By making the initial conductance balance with this auxiliary condenser it is possible to set the conductance dial at zero at all frequencies and thereby obtain direct conductance readings on the dial.

Equation (5) shows that for any given tuning inductance, L, the setting of the susceptance condenser, C_B, for the initial susceptance balance also varies with frequency. In order to make it possible to set initially at any point on the scale, an auxiliary condenser in parallel with the susceptance condenser is therefore necessary. In addition, because of the limited tuning range that can be obtained with a single coil, several different coils are necessary to cover the frequency range. These are selected by a switch.

1.7 Panel Layout and Complete Circuit Diagram

A panel view of the Twin-T is shown in Figure 2. The controls, plainly marked on the panel, are:

(1) A precision-type variable condenser (CAPACITANCE) used to measure susceptance components and having a dial and drum combination calibrated from 100 to 1100 μμf.

(2) An auxiliary condenser (AUX. TUNING CAP.), consisting of a bank of fixed condensers, controlled by push buttons, and a small variable condenser. This combination is in parallel with the precision condenser and is used to establish the initial susceptance balance at any chosen dial setting.

(3) A coil switch, marked with the frequency range covered by each tuning coil.

(4) A variable condenser (CONDUCTANCE) used to measure conductance components and having a dial that carries two scales, one from 0 to 100 μmhos and one from 0 to 300 μmhos.

(5) A 4-position switch used to establish scales on the conductance dial as described in paragraph 1.4. The nominal switch-position frequencies (1, 3, 10 and 30 Mc) are marked in large characters while the frequency limits between which the setting is usable are marked in smaller characters. The 4-position switch and the coil switch are jointly identified by the panel marking FREQ. RANGE.

(6) Two small variable condensers (INITIAL BALANCE), in parallel with the conductance condenser, used as coarse (APPROX) and fine (EXACT) controls to establish the initial conductance balance at a dial reading of zero.

The complete circuit diagram of the Twin-T, showing the switches and auxiliary condensers, is illustrated in Figure 3. The resistor-condenser combinations associated with the tuning coils are used to modify the tuning-coil conductances so that their variations with frequency do not exceed the adjustment range of the auxiliary condensers used to establish the initial conductance balance.

6. In terms of the series resistance, R, and inductance, L, the conductance is given by $\frac{R}{R^2 + (\omega L)^2}$. For values of the storage factor, Q, over 10 this is practically equal to $\frac{1}{RQ^2}$.

2.0 OPERATION

2.1 Generator

Any well-shielded radio-frequency oscillator having an output voltage of the order of 1 to 10 volts and adequate frequency stability will serve as generator.

2.2 Detector

Any well-shielded radio receiver having a sensitivity of the order of 1 to 10 μv will serve as detector. It is recommended that the receiver used be provided with an adequate r-f sensitivity control and a local oscillator to give a heterodyne note at the intermediate frequency, and a switch to cut out the avc. Most so-called "communications receivers" fill all these requirements.

2.3 Cables and Terminals

Two single-conductor coaxial cables are supplied with the instrument for connection to the generator and detector. One of these is provided with General Radio Type 774-M Cable Jacks at each end and is intended for use with a General Radio Type 605-B Standard-Signal Generator as generator. The other is provided with a Type 774-M Cable Jack at one end and spade terminals at the other. It is intended for use with any receiver having machine-screw terminals for antenna and ground. If possible, however, it is recommended that this second cable also be terminated in a Type 774-M Cable Jack and that a Type 774-G Panel Plug, into which it can be plugged, be installed at the receiver.

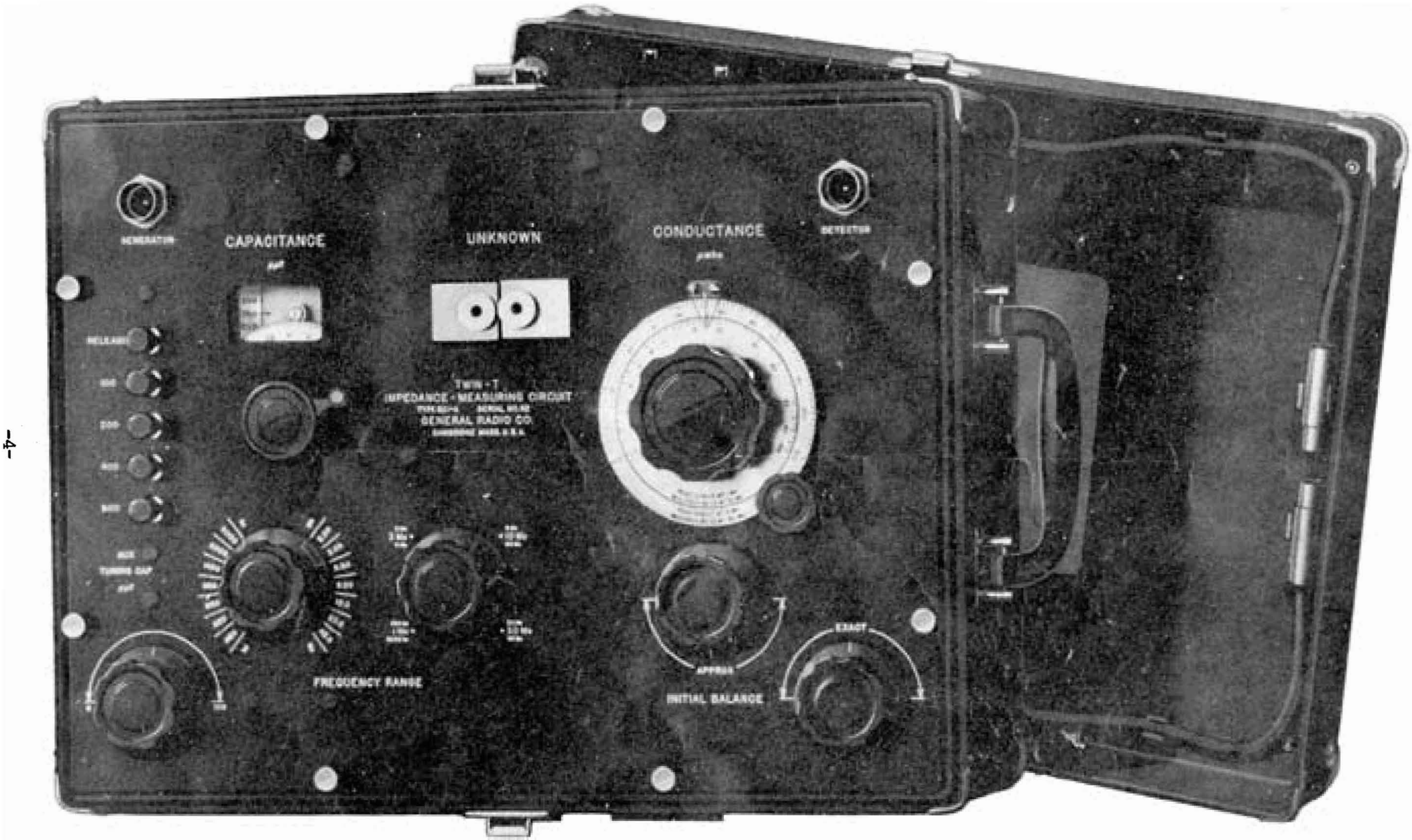

FIGURE 2. Panel view of Type 821-A Twin-T Impedance-Measuring Circuit with Cover Removed

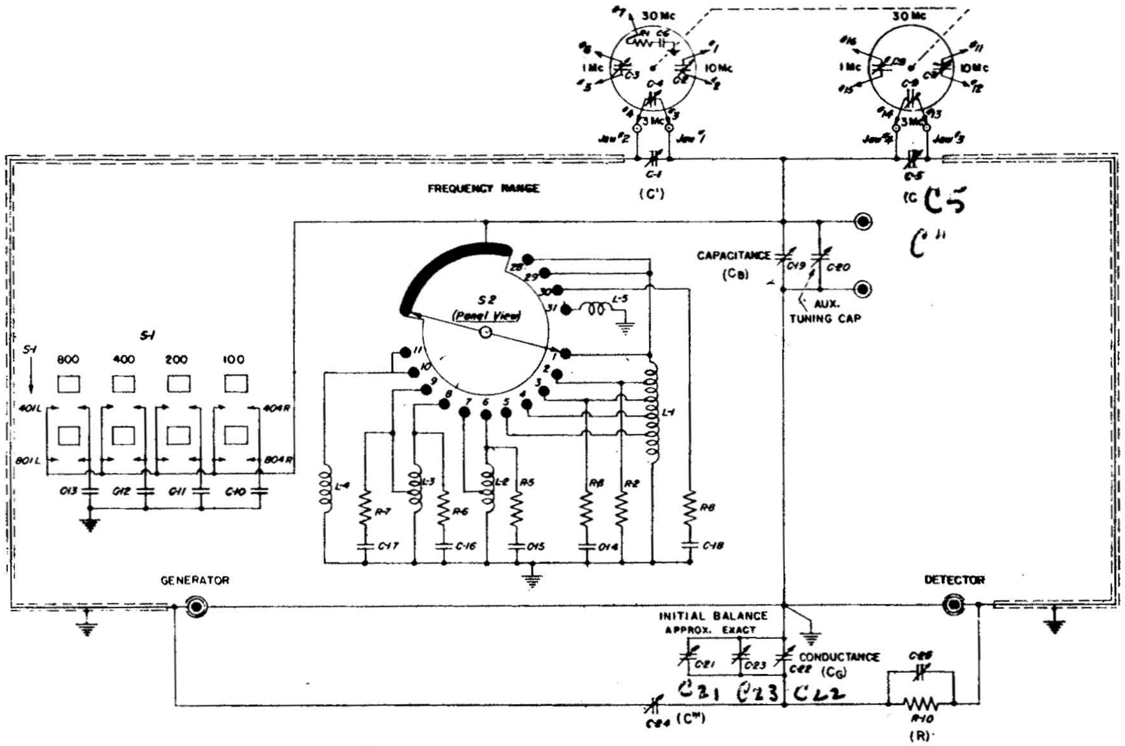

FIGURE 3. Wiring Diagram of Type 821-A Twin-T Impedance-Measuring Circuit

PARTS LIST

Condensers

C-1 = 25 μμf
C-2 = 25 μμf
C-3 = 50 μμf
C-4 = 25 μμf
C-5 = 25 μμf
C-6 = 130 μμf
C-7 = 25 μμf
C-8 = 50 μμf
C-9 = 25 μμf
C-10 = 100 μμf
C-11 = 200 μμf
C-12 = 400 μμf
C-13 = 800 μμf
C-14 = 50 μμf
C-15 = 100 μμf
C-16 = 60 μμf
C-17 = 150 μμf
C-18 = 250 μμf
C-19 = 100-1100 μμf
C-20 = 100 μμf
C-21 = 50 μμf
C-22 = 15 μμf
C-23 = 15 μμf
C-24 = 25 μμf
C-25 = 25 μμf

Resistors

R-1 = 15 Ω
R-2 = 0.1 MΩ
R-3 = 1000 Ω
R-4 =
R-5 = 150 Ω
R-6 = 100 Ω
R-7 = 50 Ω
R-8 = 15 Ω
R-9 =
R-10 = 100 Ω

Inductors

L-1 = 821-306
L-2 = 821-307
L-3 = 821-308
L-4 = 821-825
L-5 = 821-822

Switches

S-1 = 821-35
S-2 = 821-305

A special coaxial adapter (Type 774-V) is available for the Type 684-A Modulated Oscillator that will receive the Type 774-M Cable Jack and it is recommended that this be used, rather than the Type 138-V Binding Posts normally provided, if this instrument is to be used as generator.[7]

2.4 Grounding

The instrument should, in general, be grounded at a single point, through as low reactance a connection as possible. To facilitate making this connection a ground clamp is provided on the instrument case, as shown in Figure 4.

The ground lead should preferably be made with a short length of copper strip, say 1 inch wide. In laboratory setups a satisfactory "ground" can be obtained by covering the top of the bench with copper foil, even though the bench is physically far removed from ground. If the foil area is large enough, it will usually be found that a connection from it to ground, say through a steam radiator system, will make no appreciable difference in results.[8] In field setups the best "ground" is usually found to be some large metal structure, such as a relay rack.

If the grounding is not adequate it will usually be found that the panel of the instrument is at a different potential from the hand of the operator and that the balance can be changed by touching the panel, and erroneous results will be obtained.

2.5 Stray Pickup

If the panel of the instrument is at ground potential but those of the detector and generator are not it is usually an indication of excessive reactance in the connections from the outer conductors of the coaxial leads to those panels. The use of Type 774 Coaxial Connectors, as recommended in paragraph 2.3, will gen-

7. It has been found that a low-reactance connection between the outer conductor of the coaxial cable and the generator panel is very important. On this count, the combination of a Type 274-ND Shielded Plug and Type 138-V Binding Posts has been found inadequate at frequencies over about 15 Mc even though the shielding is satisfactory. If another type of oscillator is to be used as generator it is strongly recommended that a Type 774-G Panel Plug be installed to receive the Type 774-M Cable Jack.

8. The foil area should be at least great enough so that generator, Twin-T, and detector can all be placed upon it.

Figure 4.

erally eliminate these potential differences. A further test for the existence of this condition can be made by removing the detector cable from the panel jack of the Twin-T. The detector pickup should be negligibly small if the generator is adequately shielded. If the outer shell of the Type 774-M Cable Jack can then be touched to the ground post of the Twin-T without significantly increasing the receiver output, no excessive reactance exists.

If the detector, when disconnected from the Twin-T, shows considerable pickup, it is usually an indication of poor shielding in the generator or of energy transfer from the generator to detector through the power line.

It is sometimes found, in field setups where grounding conditions cannot be carefully controlled, that individual ground connections from the panels of the generator, Twin-T, and detector to a common ground point will give less pickup and more consistent results than a single common ground to the Twin-T alone. The use of coaxial connectors at both generator and detector is particularly recommended for these field setups to avoid, as much as possible, the necessity for such multiple ground connections.

2.6 Initial Balance

To place the instrument in operation, first connect the generator and detector with the cables provided in the cover and ground the instrument as described in paragraph 2.4. Next set the coil switch and the 4-position conductance switch to frequency ranges bracketing the operating frequency. Set the susceptance condenser (CAPACITANCE) to some convenient value and the conductance condenser (CONDUCTANCE) to zero. Balance to a null by varying the auxiliary condenser combination in parallel with the susceptance condenser (AUX. TUNING CAP.) and the auxiliary condensers in parallel with the conductance condenser (APPROX. and EXACT).

Figure 5 is a plot of the frequency variation of the total tuning capacitance (sum of capacitance of auxiliary condenser combination and susceptance condenser) required for the initial susceptance balance for the different coils. The plot will be found useful both in estimating the approximate initial settings and the capacitance range over which the precision condenser can be varied. At the low-frequency end of each coil range the precision condenser cannot generally be set at initial low-capacitance readings, and at the high-fre-

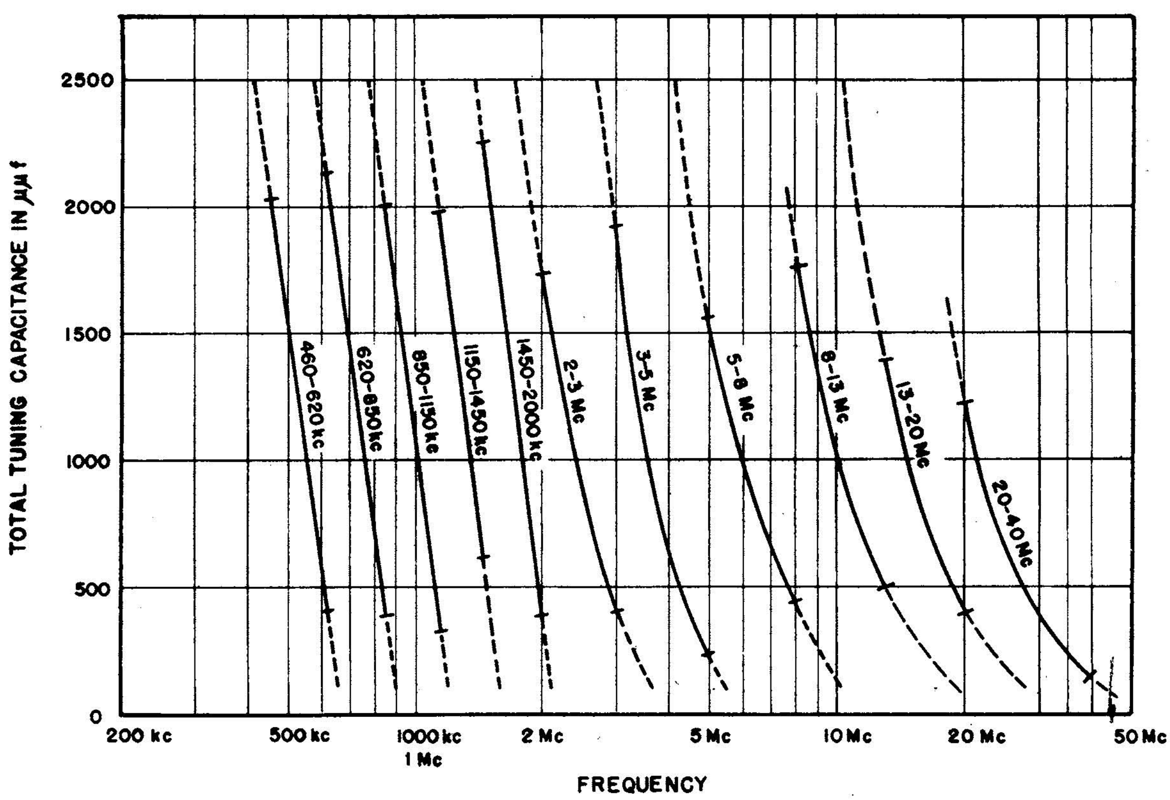

Figure 5.

quency end of each coil range it cannot generally be set at initial high-capacitance readings. By selecting the proper coil, however, it is possible to set at both minimum and maximum capacitance at any frequency in the operating range.[9]

For the detector it is particularly desirable to use a receiver that has a good r-f sensitivity control and a switch to disconnect the avc. If the receiver gain is set too high there is a tendency for the receiver output to increase as balance is approached, and if the conductance balance is not set approximately correctly it becomes quite difficult to find the susceptance balance, or vice versa. When the r-f sensitivity control is set to minimum sensitivity, and the avc is disconnected, no difficulty should be found in making the initial balance. As balance is approached, the receiver sensitivity can be increased to improve the precision of setting. For the first rough balance the generator signal can be modulated and the receiver beat oscillator turned off. The precise balance, however, should be made with the generator signal unmodulated. The avc should be left disconnected at all times. If an adequate r-f sensitivity control on the receiver is not available it is sometimes possible to accomplish the same general results by reducing the generator output, rather than the receiver sensitivity. For the precise balance the generator output should preferably be set at maximum so that the ratio of useful output to leakage is as great as possible.

Once the initial balance has been obtained, the setting of the precision condenser can be changed to any desired value and the susceptance balance reestablished by varying the auxiliary condenser combination. The choice of the initial setting depends upon the sign of the unknown susceptance that is to be measured. If it is capacitive, the initial setting should be high so that a decrease in setting can be observed when the unknown admittance is connected to the instrument; if it is inductive, the initial setting should be low so that an increase in setting can be observed when the unknown admittance is connected to the instrument.

2.7 Measurement of Unknown Admittance

2.71 Unknown Admittance Components Within Direct-Reading Ranges of Twin-T

Since a parallel-substitution method is used, the Twin-T is generally adapted to the measurement of high impedances or, specifically, to the measurement of admittances having small conductive components. In this class fall, generally, the admittances of such elements as coils, condensers, dielectric samples, antennas and unterminated transmission lines near half-wave resonance, parallel-resonant circuits and high-resistance units.

To measure admittances of this class, first establish an initial balance as described in paragraph 2.6. Then connect the unknown admittance and rebalance with the susceptance condenser, C_B, (CAPACITANCE) and the conductance condenser, C_G, (CONDUCTANCE). If C_{B_1} and C_{B_2} are the initial and final readings of the susceptance condenser, the effective parallel capacitance, C_{P_x}, of the unknown admittance is given by

$$C_{P_x} = C_{B_1} - C_{B_2} \qquad (5b)$$

and the susceptance, B_x, is given by

$$B_x = \omega (C_{B_1} - C_{B_2}) \qquad (5a)$$

If the final capacitance setting, C_{B_2}, is greater than the initial capacitance setting, C_{B_1}, the effective parallel capacitance is negative and the susceptance is inductive. If the final capacitance setting, C_{B_2}, is less than the initial capacitance setting, C_{B_1}, the effective parallel capacitance is positive and the susceptance is capacitive.

The unknown conductance, G_x, is determined from the final setting of the conductance dial, G_2.[10] If the measurement is made at a frequency of 1, 3, 10 or 30 Mc, where the dial is direct reading, the unknown conductance is directly given by the final dial reading. If it is made at any other frequency, the dial reading must be multiplied by the square of the ratio of the frequency used to the nominal switch-position frequency corresponding to the setting of the 4-position conductance range switch (see paragraph 1.4).

The result, when measured in this way, is of course in terms of the admittance, $Y_x = G_x + jB_x$. In many cases it is preferable to express it in terms of the impedance, $Z_x = R_x \pm jX_x$. This can be obtained from the relations

$$R_x = \frac{G_x}{G_x^2 + B_x^2} \qquad (2)$$

9. Because of errors caused by residual parameters, discussed in Section 3.2, the full range of the condenser cannot be used at frequencies above 20 Mc (Footnote 4). At the highest frequencies it will be found that the inductance in the leads to the auxiliary condensers causes their effective capacitances to increase above their nominal values. Since they play no part in the measurement process other than to establish an initial balance, however, this variation has no bearing on accuracy of measurement.

10. It is actually determined by the difference between the final and initial settings. The initial setting, G_1, however, is made at a setting calibrated as zero. (See paragraph 2.6).

$$= \frac{1}{G_x} \frac{1}{1 + (\frac{B_x}{G_x})^2} = \frac{1}{G_x} \frac{1}{1 + Q_x^2} \qquad (2a)$$

$$X_x = \frac{-B_x}{G_x^2 + B_x^2} \qquad (3)$$

$$= - \frac{1}{B_x} \frac{1}{1 + (\frac{G_x}{B_x})^2} = - \frac{1}{B_x} \frac{1}{1 + D_x^2} \qquad (3a)$$

For coils, the result is conveniently expressed in terms of the effective parallel inductance[11], L_{P_x}, and the storage factor, Q_x, which can be found from

$$L_{P_x} = - \frac{1}{\omega B_x} = - \frac{1}{\omega^2 C_{P_x}} \qquad (6)$$

$$Q_x = \left| \frac{B_x}{G_x} \right| \qquad (7)$$

For condensers, the result is conveniently expressed in terms of the effective parallel capacitance, C_{P_x}, and the dissipation factor, D_x, given by

$$C_{P_x} = C_{B_1} - C_{B_2} \qquad (5b)$$

$$D_x = \left| \frac{G_x}{B_x} \right| \qquad (8)$$

2.72 Unknown Admittance Components Outside Direct-Reading Ranges of Twin-T

At the sacrifice of the direct-reading features of the Twin-T, measurements can also be made of low impedances or, specifically, admittances having large conductive components. In this class fall, generally, the admittances of such elements as terminated transmission lines, antennas and unterminated transmission lines near quarter-wave resonance, series-resonant circuits and low-resistance units.

Measurements of admittances of this class are made by connecting in series with the unknown admittance an auxiliary condenser of such reactance that the net admittance of the combination falls within the direct-reading ranges of the Twin-T.[12] From measurements of the net admittance components and a separate measurement of the reactance of the auxiliary condenser it is then possible to determine the unknown impedance.

To determine the proper auxiliary capacitance to use, first establish an initial balance as described in paragraph 2.6. Next connect a small fixed condenser in series with the ungrounded lead of the unknown admittance, connect the combination to the UNKNOWN terminals of the Twin-T, and rebalance with the susceptance condenser (CAPACITANCE) and the conductance condenser (CONDUCTANCE). If the auxiliary series capacitance is too large the balance will be found to be outside the range of one of the Twin-T condensers, usually that of the conductance condenser. If it is too small, the settings for balance will not change by a sufficient amount to yield adequate precision of measurement. Change the auxiliary series condenser until a capacitance value is found that will give settings for the final balance sufficiently different from those for the initial balance to insure adequate precision. Be particularly sure to obtain a substantial change in the conductance setting.

The optimum value for the auxiliary series capacitance having been determined, the measurement procedure is as follows:

First, connect one side of the auxiliary series condenser to the ungrounded UNKNOWN terminal of the Twin-T and, with the other side of the series condenser disconnected, establish an initial balance as outlined in paragraph 2.6. The series condenser should be physically as small as possible and should be supported by its own ungrounded lead in a position as nearly the same as that which it will take when connected to the unknown impedance.[13]

Next connect the free lead of the condenser to the grounded UNKNOWN terminal and rebalance with the susceptance and conductance condensers. From the measured admittance, $Y' = G' + jB'$, the impedance, $Z' \doteq R' + jX'$, can be determined exactly by equations (2) and (3). In

11. This is the inductance that has a reactance equal to that of the tuning capacitance in a parallel-resonant circuit. For a coil having a storage factor, Q_x, of 10 or greater it is within 1% of the value of the effective series inductance, L_{S_x}. The relation between them is

$$L_{S_x} = \frac{L_{P_x}}{1 + (1/Q_x)^2} = \frac{L_{P_x}}{1 + D_x^2}$$

12. The conductance and susceptance of the combination are found, by rearranging equations (2a) and (3a), to be:

$$G = \frac{1}{R} \frac{1}{1 + Q^2} \qquad (2b)$$

$$B = - \frac{1}{X} \frac{1}{1 + D^2} \qquad (3b)$$

As the auxiliary condenser is made smaller and smaller, X and Q increase and D decreases. In the limit, therefore, as X_x approaches infinity,

$$G \rightarrow \frac{1}{RQ^2} = \frac{R}{X^2}$$

$$B \rightarrow - \frac{1}{X}$$

13. Small fixed mica condensers, such as the Cornell-Dubilier Type 5-W, are excellent for this service. See Paragraph 3.12 for a more extensive discussion of the precautions to be observed.

most cases, however, it will be found that the conductance component is negligible or, in any event, so small that the following simple approximate equations can be used:

$$R' = \frac{G'}{(B')^2} \qquad (9)$$

$$X' = -\frac{1}{B'} \qquad (10)$$

The position of the auxiliary series condenser should be changed as little as possible when this connection is made, preferably only the free lead being bent so as to make contact with the grounded UNKNOWN terminal.

Finally, disconnect the free lead of the auxiliary condenser from the grounded UNKNOWN terminal, and connect it to the ungrounded terminal of the unknown impedance. Rebalance with the susceptance and conductance condensers. From the measured admittance, $Y'' = G'' + jB''$, of the combination the impedance, $Z'' = R'' + jX''$, can be determined from equations (2) and (3):

$$R'' = \frac{G''}{(G'')^2 + (B'')^2} \qquad (2)$$

$$X'' = \frac{-B''}{(G'')^2 + (B'')^2} \qquad (3)$$

The unknown impedance, $Z_X = R_X + jX_X$, is equal to the difference between these two impedances.

$$R_X = R'' - R' \qquad (11)$$

$$X_X = X'' - X' \qquad (12)$$

The ground terminal of the unknown impedance can be left connected to the grounded UNKNOWN terminal at all times for this measurement.

It is often found that the unknown reactance, X_X, is quite small compared with the reactance, X', of the auxiliary series condenser and that the arithmetic used in its evaluation therefore involves taking the difference between two large numbers. To avoid, as much as possible, inaccuracy in slide-rule computations it is therefore helpful to express the unknown reactance, X_X, in terms of the difference between the two measured parallel capacitances, C_P'' and C_P', which can be read from the precision condenser scale.[14]

14. If the same initial setting is used for both measurements, $C_{B_1}' = C_{B_1}''$, then $C_P'' - C_P' = (C_{B_1}'' - C_{B_2}'') - (C_{B_1}' - C_{B_2}') = C_{B_2}' - C_{B_2}''$.

$$X_X = \frac{1}{B'} \cdot \frac{\frac{B'' - B'}{B''} + \left(\frac{G''}{B''}\right)^2}{1 + \left(\frac{G''}{B''}\right)^2} \qquad (13)$$

$$= \frac{1}{\omega C_P'} \cdot \frac{\frac{C_P'' - C_P'}{C_P''} + \left(\frac{G''}{\omega C_P''}\right)^2}{1 + \left(\frac{G''}{\omega C_P''}\right)^2} \qquad (13a)$$

The dissipation factor, $D'' = \frac{G''}{B''}$, of the series combination is usually small and its square can be neglected in comparison with unity. For most conditions, therefore, equation (13a) can be used in the simpler, approximate form:

$$X_X = \frac{1}{\omega C_P'}\left[\frac{C_P'' - C_P'}{C_P''} + \left(\frac{G''}{\omega C_P''}\right)^2\right] \qquad (13b)$$

Since this equation involves directly the small difference in final capacitance settings the accuracy of the result is of the same order as that of setting and reading capacitance difference.

When the unknown reactance, X_X, is small compared with the reactance, X', of the series auxiliary condenser it is also possible to simplify, to some extent, the determination of the unknown resistance, R_X. Since X' and X'' are then not greatly different, loss in the auxiliary condenser contributes practically the same conductance component, both in the measurement of the condenser alone and in the measurement of the series combination. It is therefore generally safe to neglect the condenser loss and to make the conductance balance, when measuring the condenser alone, with the EXACT INITIAL BALANCE conductance control, leaving the CONDUCTANCE dial set at zero. The conductance balance for the series combination is then made with the same setting of the INITIAL BALANCE conductance controls, so that, to a first approximation, the loss in the series condenser is allowed for in the measurement process. The unknown impedance then becomes:

$$R_X = R'' = \frac{G''}{(G'')^2 + (\omega C_P'')^2} \qquad (2)$$

$$X_X = \frac{1}{\omega C_P'}\left[\frac{C_P'' - C_P'}{C_P''} + \left(\frac{G''}{\omega C_P''}\right)^2\right] \qquad (13b)$$

2.73 Illustrative Examples

As a guide to the practical application of the material of paragraphs

2.71 and 2.72, three illustrative examples follow.

(a) Measurement of a 500-μμf Condenser at 10 Mc

Set the 4-position conductance range switch at 10 Mc and the coil switch on the 8 - 13 Mc range. Set the susceptance condenser dial at some high value, say C_{B_1} = 1000.0 μμf, and the conductance dial at zero. Adjust to an initial balance as described in paragraph 2.6. (From Figure 5 the AUX. TUNING CAP will be about 100 μμf.)

Connect the condenser to be measured across the UNKNOWN terminals and, with the susceptance and conductance condensers, adjust to a final balance. Let:

$$C_{B_2} = 442.4\ \mu\mu f$$

$$G_2 = 80\ \mu mho$$

Then:

$$C_{P_x} = 1000.0 - 442.4 = 557.6\ \mu\mu f \qquad (5b)$$

$$G_x = G_2 = 80\ \mu mho \text{ (See par. 2.71)}$$

$$D_x = \frac{80 \times 10^{-6}}{2\pi \times 10 \times 10^6 \times 557.6 \times 10^{-12}} = 0.0023 = 0.23\% \qquad (8)$$

(b) Measurement of 1 μh Coil at 25 Mc

Set the 4-position conductance range switch at 30 Mc and the coil switch on the 20.0 - 40.0 Mc range. Set the susceptance condenser dial at some low value, say C_{B_1} = 100.0 μμf, and the conductance dial at zero. Adjust to an initial balance as described in paragraph 2.6. From Figure 5, the AUX. TUNING CAP will be about 600 μμf. It will, however, appear less because of lead inductance (see Footnote 9, par. 2.6).

Connect the coil to be measured across the UNKNOWN terminals and adjust to a final balance as before. Let:

$$C_{B_2} = 139.8\ \mu\mu f$$

$$G_2 = 90\ \mu mho \text{ (Dial reading)}$$

Then:

$$B_x = 2\pi \times 25 \times 10^6\ (100.0 - 139.8) \times 10^{-12} \times 10^6 = -6250\ \mu mho \qquad (5a)$$

$$G_x = 90 \times \left(\frac{25}{30}\right)^2 = 62.5\ \mu mho \text{ (see par. 2.71)}$$

$$L_{P_x} = -\frac{10^6}{2\pi \times 25 \times 10^6 \times (-6250) \times 10^{-6}} = 1.02\ \mu h \qquad (6)$$

$$Q_x = \frac{6250}{62.5} = 100 \qquad (7)$$

(c) Measurement of Matched 72-ohm Coaxial Transmission Line at 830 kc

Set the 4-position conductance range switch at 1 Mc and the coil switch on the 620 - 850 kc range. Set the susceptance condenser dial at some value near mid-scale and the conductance dial at zero. Adjust to an initial balance as described in par. 2.6. From Figure 5, the AUX. TUNING CAP will be set at zero.

Following the procedure outlined in paragraph 2.72, find the largest convenient value of the auxiliary series condenser that will give a conductance balance on scale. Suppose it is 150 μμf, nominal value.

Measure the capacitance of the auxiliary condenser and balance with the conductance condenser set to zero with the auxiliary condenser across the UNKNOWN terminals. Disconnect the lead of the auxiliary condenser from the grounded UNKNOWN terminal, connect to the ungrounded terminal of the unknown impedance and rebalance. Let:

$$C'_{B1} = 500.0\ \mu\mu f$$

$$C'_{B2} = 352.5\ \mu\mu f$$

G'_2 = 0 μmho (Set with EXACT INITIAL BALANCE conductance control)

$$C_{B_1}'' = 500.0\ \mu\mu f$$

$$C_{B_2}'' = 353.6\ \mu\mu f$$

$$G_2'' = 60.8\ \mu mhos\ \text{(Dial reading)}$$

$$= 60.8 \times \left(\frac{0.83}{1}\right)^2 = 41.9\ \mu mho\ \text{(See par. 2.71)}$$

Then:

$$C_p' = 500.0 - 352.5 = 147.5\ \mu\mu f$$

$$G'' = G_2'' = 41.9\ \mu mho$$

$$C_p'' = 500.0 - 353.6 = 146.4\ \mu\mu f$$

$$B'' = 2\pi \times 830 \times 10^3 \times 146.4 \times 10^{-12} \times 10^6 = 764\ \mu mho$$

$$R_x = \frac{41.9 \times 10^{-6}}{(41.9^2 + 764^2) \times 10^{-12}} = 71.6\ \Omega$$

$$X_x = \frac{10^{12}}{2\pi \times 830 \times 10^3 \times 147.5} \left[\frac{146.4 - 147.5}{146.4} + \left(\frac{41.9}{764}\right)^2\right] \qquad (13b)$$

$$= -6\ \Omega \quad \text{(Capacitive)}$$

$$Z_x = 71.6 - j6$$

2.74 Balanced Lines and Antennas

The measurement of three-terminal devices, such as balanced lines and antennas, can be made with the Twin-T, although the computations involved are quite laborious.

The method depends upon the analysis of the unknown impedance in terms of the equivalent circuit of Figure 6 and requires three separate measurements, as follows:

(1) Short-circuit impedance Z_1 by grounding line A at point of measurement, and measure impedance, Z', from line B to ground.

$$Z' = \frac{Z_2 Z_3}{Z_2 + Z_3} \qquad (14)$$

(2) Short-circuit impedance Z_2 by connecting line A to line B at point of measurement, and measure impedance, Z'', from the junction to ground.

$$Z'' = \frac{Z_3 Z_1}{Z_3 + Z_1} \qquad (15)$$

(3) Short-circuit impedance, Z_3, by grounding line B at point of measurement, and measure impedance, Z''', from line A to ground.

$$Z''' = \frac{Z_1 Z_2}{Z_1 + Z_2} \qquad (16)$$

Combining equations (14), (15) and (16) gives:

$$Z_1 = \frac{2Z'Z''Z'''}{Z'Z'' - Z''Z''' + Z'''Z'} = \frac{2}{-\frac{1}{Z'} + \frac{1}{Z''} + \frac{1}{Z'''}} \qquad (17)$$

$$Z_2 = \frac{2Z'Z''Z'''}{Z'Z'' + Z''Z''' - Z'''Z'} = \frac{2}{\frac{1}{Z'} - \frac{1}{Z''} + \frac{1}{Z'''}} \qquad (18)$$

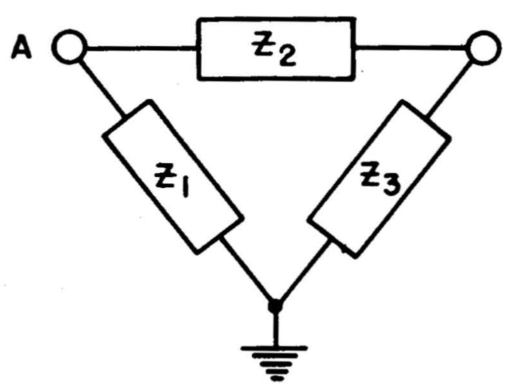

Figure 6.

$$Z_3 = \frac{2Z'Z''Z'''}{-Z'Z'' + Z''Z''' + Z'''Z'} = \frac{2}{\frac{1}{Z'}+\frac{1}{Z''}-\frac{1}{Z'''}} \quad (19)$$

This method gives each component of impedance, detecting any unbalance. At perfect balance, $Z_1 = Z_3$, $Z' = Z'''$.

$$Z_1 = Z_3 = 2Z'' \quad (17a)$$

$$Z_2 = \frac{2Z'Z''}{2Z'' - Z'} = \frac{1}{\frac{1}{Z'} - \frac{1}{2Z''}} \quad (18a)$$

When the balanced line is fed from a balanced source, the effective input impedance is given by

$$Z_{AB} = \frac{2Z_1Z_2}{2Z_1 + Z_2} = \frac{4Z'Z''}{4Z'' - Z'} \quad (20)$$

Z_{AB} is the input impedance seen from the source. It should be measured once with the far end of the line open and once with it closed if it is desired to compute the characteristic impedance and propagation constant by the usual method. No grounds should be made to the line at any point other than the input when making measurements.

The component impedances involved must usually be measured by the method described in paragraph 2.72. In equations (17) to (20) they must, of course, be written in their complex forms.

3.0 CORRECTIONS

3.1 Lead Corrections

In common with other types of impedance-measuring equipment, the Twin-T can only measure impedance at its own terminals. The residual impedances of the leads used to connect the unknown impedance to these terminals, however, often causes this impedance to differ from the impedance appearing at the terminals of the device under test. Under some circumstances the difference can be ignored and the measured impedance taken as the impedance of the device under test, including the leads. In most cases, however, the device will not be used with the same leads used to connect it to the measuring instrument and it is necessary to compensate for the effect of these leads to obtain the desired impedance. An exact correction for the effect of the leads requires analysis as a transmission line and is laborious and cumbersome. For specific measurements, however, approximate corrections will yield satisfactory accuracy.

3.11 Corrections for admittance measurements within direct-reading ranges of Twin-T

When the Twin-T is used to measure admittances within its direct-reading range the unknown impedance is so high that for relatively short leads the voltage drop along the leads is small compared with the voltage drop across the unknown impedance. The effective capacitance between the leads is consequently not materially changed when the unknown impedance is connected and disconnected. If the initial balance is established with the leads to the unknown in place, but disconnected at the far end, the lead capacitance therefore cancels out in the measurement and the only correction that need be made is for the inductive reactance.

The "shortness" of the leads is expressed by the ratio of their inductive reactance to the unknown impedance. A value of 20% for this ratio may well be taken as an upper limit. If, for instance, two parallel No. 18 B & S gauge wires, spaced 3/4" apart on centers, are used as leads, the inductance of the two wires will be about 0.037 μh/inch and the corresponding inductive reactance at 30 Mc will be about 7Ω/inch. When measuring, say, a 300Ω impedance at this frequency, then, the leads used to connect to the unknown impedance should each be less than 9" long.

The most straightforward method of correcting for the lead inductance is to convert the measured parallel admittance to the corresponding series impedance and to subtract directly from the measured reactance the inductive reactance of the leads.

The lead inductance can be determined by measuring a small fixed condenser, first at the UNKNOWN terminals of the Twin-T and then at the end of the leads. The first measured capacitance, C', with the condenser at the UNKNOWN terminals is equal to the effective parallel capacitance of the condenser; the second measured capacitance, C", with the condenser at the end of the leads is equal to $\frac{C'}{1 - \omega^2(\delta L)C'}$, where (δL) is the lead inductance. From these two measurements

$$\delta L = \frac{1}{\omega^2}\,\frac{C'' - C'}{C'C''} \quad (21)$$

As the capacitance of the fixed condenser is increased the difference between C' and C" becomes greater and the precision of measurement of δL increases.

3.12 Corrections for admittance measurements outside direct-reading range of Twin-T

The analysis presented in paragraph 3.11 applies directly, in the series-condenser method described in paragraph 2.72, to the lead from the ungrounded UNKNOWN terminal to the auxiliary series condenser, provided this lead is not made so long that there is an appreciable voltage drop along its length. The capacitance of this lead to ground is therefore automatically accounted for when the initial balance is made with it connected, but with the far end of the series condenser disconnected, as described in paragraph 2.72. The effect of the lead inductance can also be eliminated, in this method, by connecting the auxiliary condenser at the far end of the lead, where it can be connected to the ungrounded terminal of the device under test with a lead of negligible length. The far end of the condenser should then be connected to the ground terminal of the device under test, see Figure 7, rather than to the grounded UNKNOWN terminal in the Twin-T when its reactance, X', is measured. This measured reactance then in-

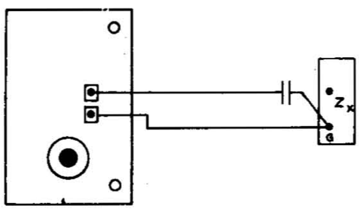

FIGURE 7.

cludes the lead reactance and, since the same lead length is used in the measurement of the series combination, the lead reactance cancels out. If the auxiliary series condenser is connected at the Twin-T end of the lead it will generally be found necessary to correct for the lead capacitance, even if the C' measurement is made by connecting the far end of the lead to the ground terminal of the device under test.

There is one other source of error, however, that should be carefully watched, when using the series-condenser method. Any condenser, when used in a series connection, will have, in addition to the direct capacitance between its terminals, capacitance from each terminal to ground. Capacitance to ground from the condenser terminal connected to the Twin-T will cause no error since it is always across the UNKNOWN terminals. Capacitance to ground from the other terminal, however, will cause the measured value of C' to differ from the direct capacitance between the terminals since this capacitance to ground is effectively across the UNKNOWN terminals for the initial balance but is shorted out when the condenser is connected across the UNKNOWN terminals. While the capacitance to ground is ordinarily very small, it is often necessary to use small series capacitances (as low as 20 - 30 μμf) and it is not difficult to produce appreciable errors. It is strongly recommended, therefore, that the series condenser used be of as small dimensions as possible. As a further safeguard it is suggested that the body of the condenser be wrapped with copper foil, connected to the lead to the Twin-T, so that the ground capacitance is all thrown over to that side of the condenser, see Figure 8, where it can cause no error.

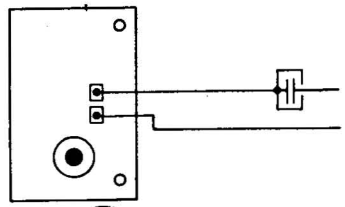

FIGURE 8. Recommended method of shielding auxiliary condenser.

3.2 Corrections for Residual Parameters

The upper-frequency limit of accurate operation of radio-frequency impedance-measuring equipment is nearly always determined by residual parameters, in the wiring and in the impedance elements, that are not accounted for in the ordinary theory of operation. While these have been made extremely small in the Twin-T, they are still large enough to affect performance at the highest frequencies and to set the maximum usable frequency in the neighborhood of 30 Mc.

By careful balancing of impedance levels and attention to mechanical arrangement, the effects of all the residual parameters except those occurring in the susceptance condenser, C_B, have been made negligible. The residual parameters in this condenser, for which corrections may be necessary, are:

(1) Inductance, L', between the condenser and the ungrounded UNKNOWN terminal.

(2) Inductance, L", between the condenser and the point in the Twin-T circuit to which it connects.

(3) Inductance, L_c, in the metal structure of the condenser itself.

(4) Resistance, R_c, in the metal structure of the condenser.

Figure 9 is an equivalent circuit showing the residual parameters listed and their relative locations. They are all essentially constant, independent of the setting of the susceptance condenser, and have the following values at a frequency of 30 Mc:

$L' = 6.8 \times 10^{-9}$ h
$L'' = 3.15 \times 10^{-9}$ h
$L_c = 6.1 \times 10^{-9}$ h
$R_c = 0.026\ \Omega$

The inductances are independent of frequency. The resistance varies directly as the square root of the frequency.

3.21 Correction for L'

The inductance, L', is directly in series with the unknown admittance. To correct for its effect it is therefore only necessary to subtract its inductive reactance from the measured reactance as described in paragraph 3.11 for lead reactance. When lead corrections are necessary, it can be taken into account as part of the lead correction by increasing the measured lead inductance by 6.8×10^{-9} h. When measuring low impedances by the series-condenser method its effect is eliminated, along with that of the lead inductance, by following the procedure outlined in paragraph 3.12.

3.22 Correction for L"

The inductance, L", has no appreciable effect upon the susceptance balance but causes the apparent conductance, measured by the conductance dial, to differ from the true value. To a first approximation the true value of the unknown conductance, G_x, is found from the measured value of unknown conductance, G_2, and the initial susceptance condenser capacitance, C_{B_1}, by

$$G_x = G_2\,(1 - \omega^2 L'' C_{B_1})^2 \qquad (22)$$

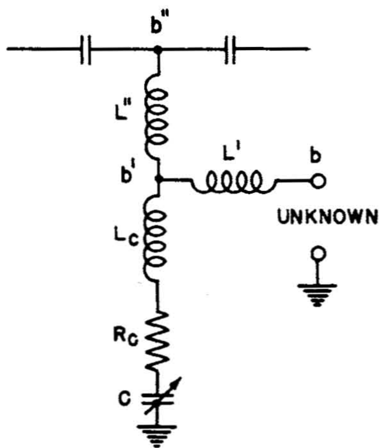

FIGURE 9. Equivalent circuit of the susceptance condenser showing residual parameters.

3.23 Correction for L_c

The inductance, L_c, causes the high-frequency effective capacitance of the susceptance condenser, C_B, to rise above the low-frequency calibration. The apparent susceptance, measured by the susceptance condenser, therefore differs from the true value. To a first approximation the true value of the unknown susceptance, B_x, is

$$B_x = \frac{\omega\,(C_{B_1} - C_{B_2})}{1 - \omega^2 L_c (C_{B_1} + C_{B_2})} \qquad (23)$$

3.24 Correction for R_c

The resistance, R_c, causes the effective conductance of the susceptance condenser, C_B, to vary as the capacitance is changed. It therefore introduces an error in conductance measurement when the unknown admittance has a relatively large susceptive component. To correct for its effect, the conductance component, δG, should be added algebraically to the measured conductance.

$$\delta G = R_c \omega B_x (C_{B_1} + C_{B_2}) \qquad (24)$$

For capacitive unknown susceptances this correction is positive, for inductive susceptances negative.[15]

3.25 Application of Corrections

The systematic application of the corrections given by equations (22) to (24) will yield results that are limited largely by the calibration accuracy of the instrument. For highest accuracy, however, it is recommended that the residual parameters be measured for the particular instrument in use.[16]

Since the corrections for errors caused by L", L_c and R_c all increase with the capacitance, C_B, of the susceptance condenser, the settings of this condenser should be kept as low as possible. For frequencies above 20 Mc the tuning coil has been so chosen that initial susceptance balances cannot be made at settings so high that excessive errors occur. When inductances are measured the final susceptance balances should be kept within the range for which initial balance is possible.

15. When measuring low-loss condensers and dielectric samples, the error is often sufficiently great to cause negative conductance readings unless the correction is applied.

16. See D. B. Sinclair, "The Twin-T, A New Type of Null Instrument for Measuring Impedance at Frequencies up to 30 Mc", Proc.I.R.E. for a complete description of methods of measurement.